Spatially average of turbulence intensity inside large wind turbine arrays

Sten Frandsen and Peter Hauge Madsen
Risø National Laboratory
4000 Roskilde
Denmark
e-mail: sten.frandsen@risoe.dk

1 Introduction

For large wind farms it is necessary to re-evaluate the concept of "ambient" turbulence intensity. In the interior of the wind farm, the standard deviation of turbulent wind speed fluctuations will be composed by a horizontal average of fluctuations, and a component representing the well defined, nearer wakes. In Frandsen and Thøgersen (2001) and Frandsen (2003), a model for the impact on fatigue loading of increased turbulence in wind turbine arrays was presented. The spatially *averaged* component termed “ambient" or average wind farm turbulence is an important component of the fatigue model and is treated in further details in here.

2 Ambient flow and average wind farm flow

Firstly, fundamental properties of the neutrally stratified planetary boundary layer are outlined and common engineering practice for free-flow standard deviation of turbulent wind speed fluctuations in the free-flow is described. And secondly, the *spatially averaged* level of standard deviation of turbulent wind speed fluctuations in “large” wind farms is evaluated.

2.1 Ambient turbulence in the free-flow

In the neutrally stratified atmosphere, the horizontal turbulent shear stress is typically assumed independent of height, $\tau = \rho u_*^2$, where ρ is the air density and u_* is the so-called friction velocity. The standard deviation of along-wind wind speed fluctuation σ_u is experimentally found to be proportional to the friction velocity u_* and it has been demonstrated that the standard deviation of wind speed fluctuations can be approximated as $\sigma_u \approx 2.5 \cdot u_*$. Together with the assumption of a logarithmic vertical wind profile this provides the following expression for standard deviation of wind speed fluctuations:

$$\frac{U}{u_*} = \frac{1}{\kappa} \ln \frac{z}{z_0} \quad \Rightarrow \quad u_* = \frac{\kappa U}{\ln(z/z_0)} \quad \Rightarrow \quad \sigma_u \approx \frac{U}{\ln(z/z_0)}, \qquad (1)$$

where U is mean wind speed, z is height over the terrain surface and z_0 is the surface roughness. The expression, Eq. (1), is adapted by the Danish standard for loading of civil engineering structures, DS410 (1998), and the standard for design of wind turbine structures, DS472 (1992). Like the friction velocity, the standard deviation of turbulent wind speed fluctuations may deviate substantially from Eq. (1), under stable and unstable stratification of the atmosphere. However, significant non-neutral atmospheric conditions mostly occur at lower wind speeds. Therefore, for extreme loading, under extreme wind conditions, it is of little consequence whether non-neutral stratification is taken into account.

In the international standard, IEC 61400-1 (1999), the design turbulence, $\sigma_{u,IEC}$, is given by the following expression:

$$\sigma_{u,IEC} = I_{15}(15 + a \cdot U)/(a+1) . \qquad (2)$$

Here, I_{15} is a characteristic value of hub-height turbulence intensity at the wind speed 15 m/s, and a is a slope parameter (not dimensionless). The expression takes into account the frequently encountered over-representation of unstable atmospheric conditions at lower wind speeds. That IEC61400-1 (1999) includes unstable weather conditions must – at least implicitly – be aimed at fatigue-related load cases.

The IEC 61400-1 (1999) operates with two turbulence levels, where for "low" turbulence $(I_{15};a) = (0.16;3)$ and $(I_{15};a) = (0.18;2)$ for "high" turbulence. The expression is the best estimate of the average standard deviation of turbulent wind speed fluctuations as experienced/measured in nature, plus one standard deviation of the same quantity. The standard deviation, denoted $\Delta\sigma_u$, of σ_u is specified as

$$\Delta\sigma_u = D\{\sigma_u\} = 2I_{15} . \qquad (3)$$

A little unfortunate, the constant "2" has the dimension m/s. Thus, the coefficient of variation for the standard deviation of turbulent wind speed fluctuations is

$$\delta = \frac{\Delta\sigma_u}{\sigma_{u,IEC} - \Delta\sigma_u} . \qquad (4)$$

This quantity is plotted in Figure 1, for the "low turbulence" case. When wind speed increases from 10 to 20 m/s, the coefficient of variation decreases from approx. 20% to 10%.

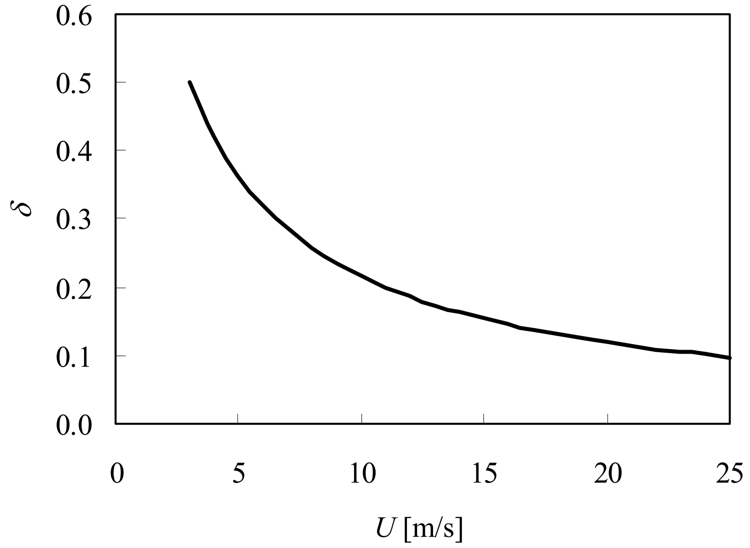

Figure 1 Coefficient of variation for standard deviation of turbulent wind speed fluctuations as function of wind speed, following IEC61400-1 (1999).

Thus,

$$\sigma_{u,IEC} = \overline{\sigma}_u + \Delta\sigma_u \,. \qquad (5)$$

Assuming that the standard deviation of turbulent wind speed fluctuations is normal distributed, the above value constitutes a percentile of approximately 80%, i.e. for a given mean wind speed and a given 10 min. period, there is a probability of 80% that σ_u is less than $\sigma_{u,IEC}$.

2.2 Scale(s) of turbulence

Though not being used directly in the below modelling of wind farm average turbulence, an assessment of the scale of turbulence is made in the following since the quantity is of significance for structural loading. While in principle the scales of turbulence are easily determined from time series of the three wind speed components, the scales of turbulence are usually not recorded and stored as other statistics such as mean and variance when performing measurements. The scale is determined as the integral of the auto-correlation function, as the maximum of the power spectrum or possibly as the up-crossing frequency of the mean of wind speed. All methods are sensitive to the assumption of stationarity of the time series. Or alternatively expressed, wind speed fluctuations may have significant energy at low spectral frequencies not captured by analysis of 10min series. Therefore, the uncertainty related to determination of the scale(s) of turbulence is large.

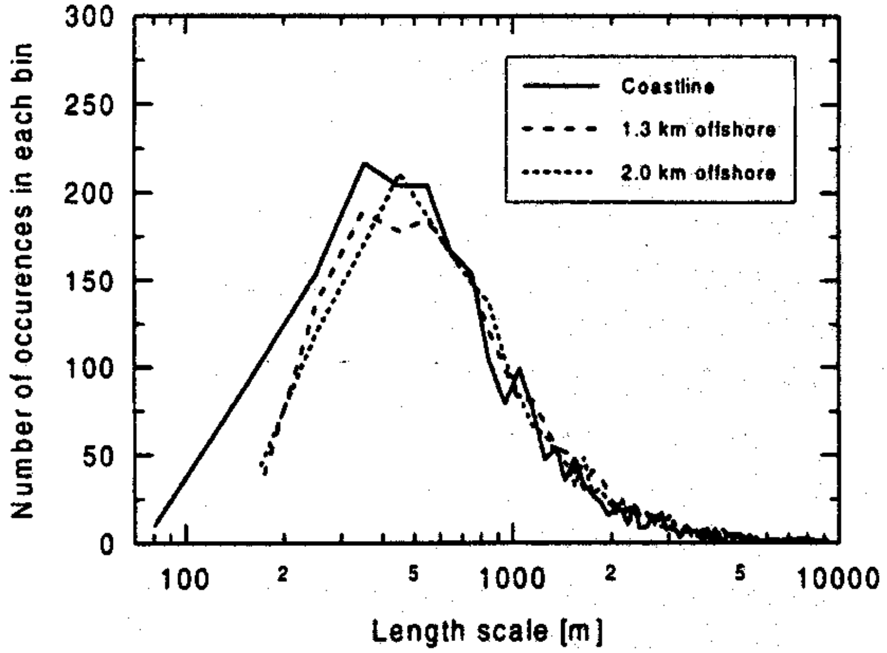

Figure 2 Measured PFD of length-scale of turbulence. From Petersen et al (1998).

Despite the problems, the PDF of turbulence length-scale[1] has been experimentally estimated, Petersen et al (1998). The result is shown in Figure 2. For the considered

---

[1] The length-scale, and not the time-scale, is usually preferred since it presumably – for a given height - is independent of mean wind speed.

data, the scale of the along-wind component of wind velocity, measured at height 48m, varies between 100 and 2000m, with a maximum at $L \approx 500$m. The variation is due to varying atmospheric stability, but also the mentioned uncertainties related to the lack of stationarity. While in the free flow, the average of length-scale is expected to be constant at a fixed height, the characteristic frequency is – by means of Taylor's hypothesis on "frozen turbulence" – linked to the mean wind speed and length-scale by

$$f \propto \frac{U}{L}. \qquad (6)$$

Thus, the frequency scale varies by a factor of two when the wind speed varies between 10m/s and 20 m/s, which are wind speeds most relevant to fatigue loading. This variation in the frequency scale is small compared to the orders of magnitude in variation of the observed turbulence scale mentioned above, and illustrated in Figure 2.

For *wake condition,* the scale of turbulence has been evaluated relative to free-flow conditions, Højstrup (1990), Crespo and Hernandez (1996) and Højstrup (1999), employing measurements and computer simulations. Both measurements and simulations showed that in the upper wake (above hub height) the scale is approximately unchanged and in the lower wake the length-scale is reduced to about half the free-flow scale. From wind tunnel measurement, Tindal et al (1993) report reductions in length-scale in the wake from 1/2 to 1/5 of the free-flow scale, depending on distance from the wind turbine. Yet another set of field measurements of length-scale in $4D_0$ distance downwind, Verheij et al (1993), report reductions to about half of free-stream conditions.

From the above it is noted that the natural variability of the free-flow scale of turbulence is large compared to the change of scale of turbulence imposed by the wakes.

Højstrup (1999) also investigated lateral and vertical coherence (corresponding, but identical to cross-wind scales of turbulence), again finding that deviations from the free-flow case are minor.

Apart from the scale of turbulence, the general observation is that the power spectral characteristics (*shapes* of spectra) in the wake are fairly well represented by the models applied for the free-flow, with the exception that wake turbulence tends to be more isotropic, i.e. the turbulence characteristics are independent of direction.

2.3 Ambient Turbulence within the Wind Farm

One or two rows of wind farms away from the edge of the wind farm, when there is not a wind turbine immediately upwind, the experienced turbulence may be expected to be identical to the free-flow turbulence. Further into the interior of the wind farm, no wind direction offers a flow that is unaffected by wind turbine wakes. Still, at any given point of observation, the standard deviation of turbulent wind speed fluctuations may be described as composed by a horizontal average of fluctuations, and a component representing the well-defined wake, generated by the wind turbine immediately upwind. The well-defined wake component is discussed in Frandsen (2003) and well as in the literature in general, whereas the average component termed "ambient wind farm turbulence" is treated here.

The mean wind speed will be reduced relative to the free-flow wind speed. The first effort to estimate the wind speed reduction in infinite clusters of wind turbines was made by Templin (1974), and others followed, see Bossanyi et al (1980), Frandsen (1992), and Emeis and Frandsen (1993).

Adopting the view that the wind turbines can be considered as roughness elements, also the general level of turbulence intensity will increase, not only under distinct wake conditions. In order to estimate the general decrease in mean wind speed and increase in turbulence intensity, consider an infinitely large wind farm. Applying a simplified version of the geostrophic drag law, Jensen (1978), the horizontally averaged, vertical wind profile down to hub height in the wind farm may be described as, see Frandsen (1992) and Frandsen (2003).

$$\frac{U_h}{u_{*0}} = \frac{1}{\kappa}\ln\left(\frac{h}{z_{00}}\right), \qquad (7)$$

where the apparent, combined roughness of the ground and the wind turbines is

$$z_{00} = h \cdot \exp\left(-\frac{\kappa}{\sqrt{c_t + \left(\kappa / \ln(h/z_0)\right)^2}}\right),\ c_t = \frac{\pi C_T}{8 s_r s_f}, \qquad (8)$$

where h is hub height, z_0 is the roughness length of the terrain surface, C_T is the wind speed dependent thrust coefficient of the wind turbines, and s_r and s_f are distances between the units in the rows and the separation between the rows, normalised with the rotor diameter[2]. The above-wind-farm friction velocity and the hub height wind speed becomes

$$u_{*0} = \frac{\kappa G}{\ln\left(\frac{G}{f' h}\right) + \frac{\kappa}{\sqrt{c_t + \left(\kappa / \ln(h/z_0)\right)^2}}}, \qquad (9)$$

and

$$U_h = \frac{G}{1 + \ln\left(\frac{G}{f' h}\right)\frac{\sqrt{c_t + \left(\kappa / \ln(h/z_0)\right)^2}}{\kappa}}, \qquad (10)$$

where G is the geostrophic wind speed and $f' \approx 1.2 \cdot 10^{-4} \cdot \exp(4) = 6.5 \cdot 10^{-3}$ at latitude 55°. Above hub height, the friction velocity is assumed height-independent.

Like for the simple free flow, σ_u is assumed proportional to the friction velocity. In the free flow – at height h – turbulent fluctuations σ_0 and turbulence intensity I_0 are:

$$\sigma_0 \approx \frac{U_0}{\ln(h/z_0)} = \frac{u_*}{\kappa},\quad I_0 = \frac{\sigma_0}{U_0}. \qquad (11)$$

Similarly, turbulence over the wind farm is estimated as

$$\sigma_{T,wf} = \frac{u_{*0}}{\kappa}. \quad (12)$$

---

[2] If the wind turbine units are located in an irregular way, then s and s_1 should be taken as averages in the wind farm.

This expression may more generally be assumed valid some distance above the wind farm. Straining the physics, it is assumed that the expression is valid all the way down to the top position of the blades. For practical reasons, the turbulence intensity in the wind farm is defined referring to free-flow hub height wind speed, U_0:

$$I_{T,wf} = \frac{\sigma_{T,wf}}{U_0}. (13)$$

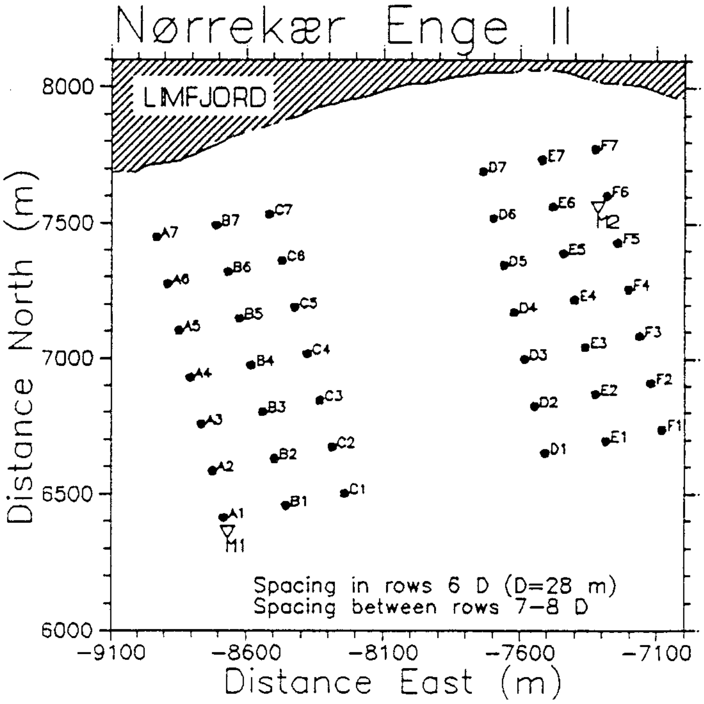

Figure 3 Layout of Nørrekær Enge II wind farm. 42 x 300kW, rotor diameter D=28m.

Experimental evidence is offered to support the model for the above-wind-farm standard deviation of turbulent wind speed fluctuations, Frandsen and Christensen (1994). Flow measurements were carried out in the Nørrekær Enge II Wind Farm, Figure 3, which consists of two groups of 3 x 7 (300 kW) Nordtank units. One met mast is placed at the Southwest edge of the wind farm and for south-westerly winds the free-flow characteristics is measured here. Another met mast is placed in the interior of the wind farm, in the north-easterly corner.

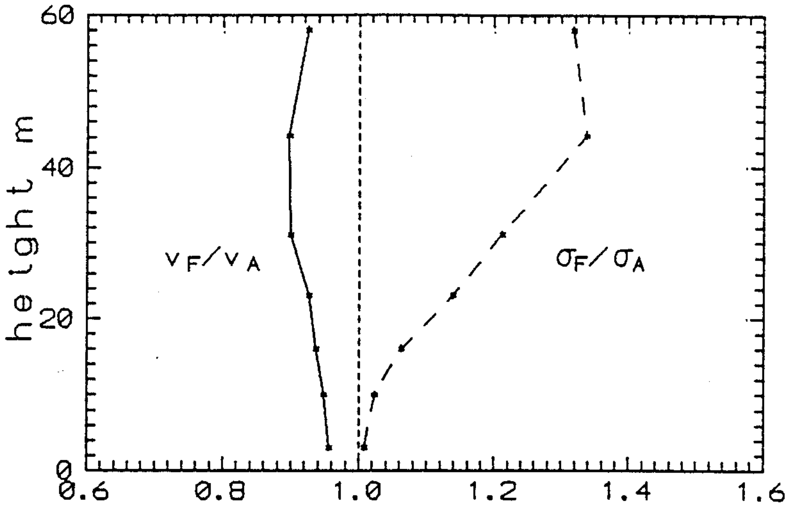

Figure 4 Ratios of wind speed (v_F/v_A) and standard deviation of turbulent wind speed fluctuations (σ_F/σ_A) profiles inside and outside the wind farm. Ambient wind speed between 8 and 9 m/s. The indeces "A" and "F" refer to met masts M1 and M2, respectively. From Frandsen and Christensen (1994).

Vertical profiles of mean wind speed and standard deviation of turbulent wind speed fluctuations were taken at both towers and averaged over a 30° wind direction sector of winds from the Southwest.

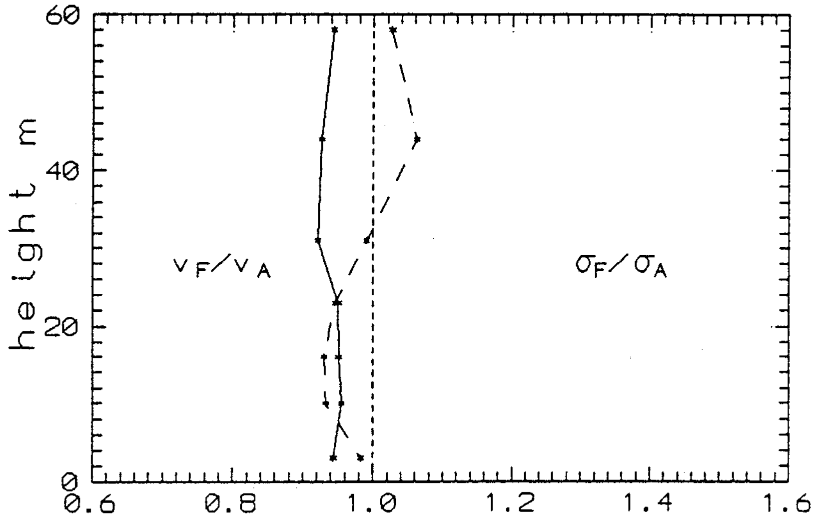

Figure 5 Ratios of wind speed and standard deviation of turbulent wind speed fluctuations vertical profiles inside and outside the wind farm. Ambient wind speed between 12 and 14 m/s. From Frandsen and Christensen (1994).

Figure 4 and Figure 5 show the ratio between the profiles of mean wind speed and standard deviation of wind speed fluctuations for 8m/s < U < 9m/s and 12 < U < 14m/s, respectively. For the low wind speed range (high C_T values) there is an increase in standard deviation of turbulent wind speed fluctuations above hub height of approx. 35% and near the ground, σ_u is unchanged. For the high wind speed the increase above hub height is approx. 10% and below "rotor height" there is an apparent decrease in σ_u of about the same magnitude. This decrease below the rotor is actually predicted by the presented model (by solving also for the friction velocity under hub height). However, in the following the standard deviation of turbulent wind speed fluctuations below hub height is, conservatively with respect to the method, assumed unchanged relative to free-flow conditions.

Figure 6 shows measured and modelled ratios of wind speed and standard deviation of turbulent wind speed fluctuations inside and outside the wind farm at hub height and at height 58m, respectively. The wind speed deficit at hub height is over-predicted by factor of two, which could indicate that in terms of mean wind speed, this wind farm is not "large".

For σ_u above rotor height, the model deviates little from the data, indicating that in terms of standard deviation of wind speed fluctuations the Nørrekær Enge II wind farm is "large". The result is encouraging, although it should be taken with caution since the specific position of the M2 met mast and the averaging over near-wake and non-wake conditions may influence the result.

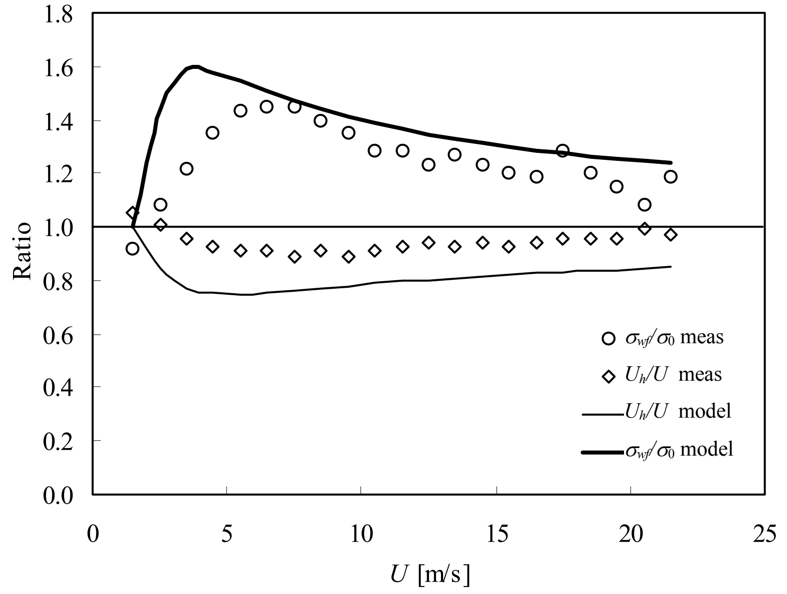

Figure 6 Ratios of wind speed at hub height (31m) and turbulence (58m) inside and outside the wind farm, as function of wind speed. The full lines are the model predictions.

The wind farm "ambient" turbulence may be decomposed in a component from terrain surface roughness and a component stemming from the presence of the wind turbines:

$$\sigma_{T,wf}^2 = \sigma_0^2 + \sigma_{addwf}^2, \quad I_{T,wf}^2 = I_0^2 + I_{addwf}^2 \Rightarrow \quad I_{addwf} = \sqrt{I_{T,wf}^2 - I_0^2}, \qquad (14)$$

where turbulence intensity refers to hub height wind speed. Eqs. (9) and (10) are fairly complex and a simplification is useful. An expression for the asymptotic value of I_{addwf} can be derived. Firstly, assuming that the contribution from terrain surface to wind farm roughness, z_{00}, is negligible, then

$$z_{00} = h \cdot \exp\left(-\frac{\kappa}{\sqrt{c_t}}\right), \qquad (15)$$

Next, an approximation to I_{addwf}, Eq (14), may be obtained:

$$I_{addwf} \approx \frac{\frac{1}{\kappa} u_{*addwf}}{U_{h,free}} \approx \frac{\ln\left(\frac{G}{f' \cdot z_0}\right)}{\ln\left(\frac{h}{z_0}\right)} \cdot \frac{1}{\ln\left(\frac{G}{f' \cdot h}\right) + 0.64 \cdot \sqrt{s_r s_f / C_T}} \qquad (16)$$

Thus, the turbulence intensity added due to the presence of the wind farm is a function of s_r, s_f and C_T, but also hub height of the wind turbines h, roughness of the terrain surface z_0 and the geostophic wind speed G. However, I_{addwf} is fairly insensitive to the geostrophic wind speed, terrain roughness and hub height and the following approximation for I_{addwf} may be adopted, Frandsen (2003):

$$I_{addwf} \approx \frac{a\sqrt{C_T}}{b\sqrt{C_T} + \sqrt{s_f s_r}} = \frac{0.36}{1 + 0.2\sqrt{s_f s_r / C_T}}, \quad \text{with } a = 1.8 \text{ and } b = 5\,. \qquad (17)$$

The approximation is tested in Figure 7. For other values of G, z_0 and h, the deviation is larger, up to a few percent. In the context, the approximation is found adequate. Also, it should be noted that the turbulence intensity in general is a function of height of observation, i.e. it will vary over the rotor. This was neglected in recognition of the general crudeness of the modelling effort and to arrive at the simple expression of Eq. (17).

Above, it was assumed that the spatially averaged shear force/stress from the wind turbines acts on the flow at hub height. Actually, the shear imposed by the wind turbines is vertically distributed from top to bottom position of the blade tips. Therefore – and with support in the data presented in Figure 4 and Figure 5 – it is stipulated that σ_u is constant above blade top position and constant below blade bottom position. And that σ_u varies linearly between these constant levels over the rotor. Thus, the average of the above-wind-farm turbulence intensity and the (chosen) below-rotor turbulence level, I_0, is applied as "wind farm ambient turbulence intensity":

$$I_0^* = \tfrac{1}{2}\left(\sqrt{I_{addwf}^2 + I_0^2} + I_0\right). \qquad (18)$$

This is thee sought model for spatially averaged turbulence intensity in wind farms.

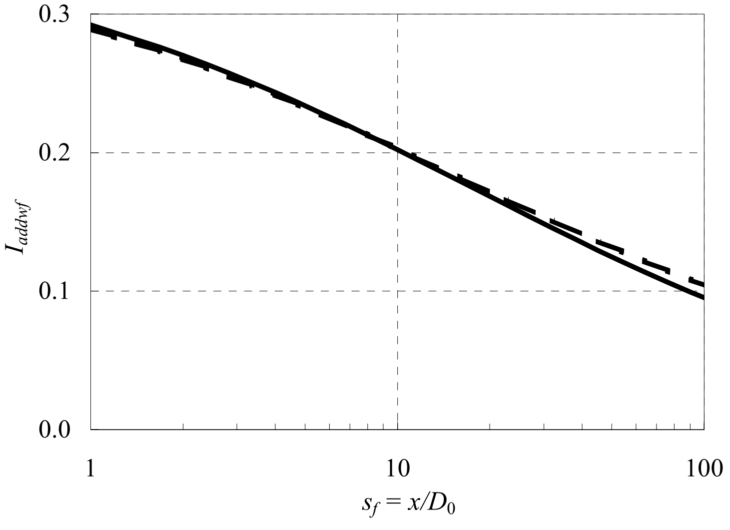

Figure 7 Horizontal average of added wind farm turbulence, as function of separation between rows. The solid line is from the basic equations, the broken line corresponds to the proposed approximation (s_r=1.5, C_T=1, h=50m, z_0=0.01m, G=15m/s).

In terms of fatigue-life consumption – when integrating over all positions of the blade – the expression tends to be slightly non-conservative. However, Eq. (18) also includes the direct-wake contributions and when combining wind farm “ambient” and individual wake turbulence, then the individual wake turbulence is to some (lesser) extent counted twice, thus countering the non-conservatism.

3 Conclusion

A model for the spatially averaged turbulence in large wind farms was presented. The model is part of a general model for turbulent fatigue loading inside wind turbine clusters.

The model is based on the general concept of turbulent boundary layer flow. In developing the model the physics is strained. This is justified by the need for a relatively simple model, by good support in data and by the general uncertainty attached to the fundamentals of wind farm flow.

4 References

Bossanyi, E.A, C. Maclean, G. E. Whittle, P. D. Dunn, N. H. Lipman and P. J. Musgrove (1980) The efficiency of wind turbine clusters, Third International Symposium on Wind Energy Systems, BHRA, Copenhagen, Denmark, August

Crespo, A. and J. Hernandez (1996) Turbulence characteristics in wind-turbine wakes, Jour. Wind Engineering and Industrial Aerodynamics 61, pp. 71-85.

XDS472 (1992) Last og sikkerhed for vindmøller (In Eng.: loads and safety for wind turbine structures), Dansk Ingeniørforening og Ingeniør-Sammenslutningen, Teknisk Forlag København, NP-209-N, 62 p.

DS410 (1998) Norm for last på konstruktioner (code of practice for loads for the design of structures) Dansk Standard 4. udgave/1. oplag, 121 p.

Emeis, S. and Frandsen, S.(1993) Reduction of horizontal wind speed in a boundary layer with obstacles, Boundary Layer Meteorology 64, pp 297-305.

Frandsen, S. (1992) On the wind speed reduction in the center of large clusters of wind turbines; Jour. of Wind Engineering and Industrial Aerodynamics, 39, pp. 251-265.

Frandsen, S. and J.C. Christensen (1994) Vindeby Offshore Windfarm - fatigue measurements; EWEC'94 Conference, Thessaloniki, Greece, October.

Frandsen, S. and M.L. Thøgersen (1999) Integrated fatigue loading for wind turbines in wind farms by combining ambient turbulence and wakes, Jour. Wind Engineering vol. 23, No. 6, pp. 327-340.

Frandsen, S. (2003) Turbulence and turbulence generated fatigue in wind turbine clusters, Risø report R-1188 (to be published).

IEC 61400-1 (1999) Wind turbine generator systems – Part 1: Safety requirements, based on 88/98/FDIS, International Electrotechnical Commission, Second edition.

Højstrup, J. (1990) Wake measurements on the Nibe wind-turbines in Denmark, Appendix 1: Data report – power spectra, Final report on CEC contract N.EN3W.0039.UK (H1)

Højstrup, J. (1999) Spectral coherence in wind turbine wakes, Jour. Wind Engineering and Industrial Aerodynamics 80, pp. 137-146.

Jensen, N.O. (1978) Change of suface roughness and the planetary boundary layer, Quart. J. R. Met. Soc, No. 104, pp. 351-356.

Petersen, E. L., N. G. Mortensen, L. Landberg, J. Højstrup and H. P. Frank (1998) Wind power meteorology, Part I: Climate and turbulence, Wind Energy, **1**, pp. 2-22.

Templin, R. J. (1974); An estimation of the interaction of windmills in widespread arrays, National Aeronautical Establishment, Laboratory Report LTR-LA-171. Otawa, Canada.

Tindal, A. J., A. D. Garrad, G. Schepers, B. Bulder, H. Hutting, F. Verheij (1993) Dynamic loads in wind farms, Proc. of European Community Wind Energy Conference, Germany, pp. 477-480.

Verheij, F. J., J. W. Cleijne, S. G. Voutsinas, S. Huberson, A. Crespo, D. Delaunay, G. Taylor (1993) Wake and wind farm modelling, Proc. of European Community Wind Energy Conference, Travemunde, Germany, pp. 400-403 .