

**On mean wind and turbulence profile measurements from ground-based wind lidars:  
limitations in time and space resolution with continuous wave and pulsed lidar systems**

**- a review**

by

Torben Mikkelsen

Wind Energy Division  
Risø National Laboratory for Sustainable Energy  
Technical University of Denmark – DTU  
DK-4000 Roskilde, Denmark

[torben.mikkelsen@risoe.dk](mailto:torben.mikkelsen@risoe.dk)

**Abstract:**

Two principal different coherent laser Doppler wind lidar anemometers have recently become available on the wind energy market for ground-based vertical mean wind and turbulence measurements. These two types of lased Doppler based wind lidars are:

1) continuous wave (cw) wind lidars

and

2) pulsed wind lidars

Although build on the same recent communication technology 1.55  $\mu$  telecom fibre technology, there nevertheless exists some fundamental differences between the lidars temporal and spatial resolution capabilities that impacts on their mean wind and turbulence measurements in the atmospheric boundary layer and therefore also of relevance for the wind energy assessment studies, and other exiting wind and turbulence research applications.

The two lidar types spatial and temporal resolution characteristics is here reviewed as described in the literature and their influence on mean and turbulence profiles as measured from ground-based lidar platforms probing the atmospheric surface layer will be discussed.

An intercomparison of the sounding volume characteristics with two specific commercially available wind lidars, one of cw type and one of pulsed type, is presented together with best estimates of these specific lidar systems optical parameters. The two specific wind lidar systems considered here are

a) the cw wind lidar ZephIR available from National Power U.K., and

b) the pulsed wind lidar named WindCube, Leosphere, Fr.

## 1. Introduction

Measurements of atmospheric wind and turbulence are always influenced by the measurement devices temporal and spatial resolution capabilities. For wind lidars, measuring in the open atmosphere, the sounding volumes are usually much bigger than the corresponding volumes associated with in-situ mast mounted anemometers (such as cups, vanes, sonics etc.)

The lidar instruments sounding volume issues are therefore of uttermost importance for a correct and unbiased data interpretation of both mean wind and turbulence quantities measured by a wind lidar.

Depending on the amount of wind shear in the probed part of the atmosphere, a wind lidar will faulty measure a biased estimate of the mean wind speed and the mean wind vertical shear profiles, if the lidars sounding volumes are not small compared to the length scales in the atmospheres shear flow and turbulence.

For instance, in order to obtain an un-biased measurement of the mean wind profile, the radial extent of a particular wind lidars sounding volume  $\Delta z$  should principally be much smaller than the mean wind shear length scale  $U(z)/(dU(z)/dz)$ , where  $U(z)$  is the vertical mean wind profile.

For interpretation of turbulence data measured by lidars the finite sounding volume effect becomes even more significant. Extensive work and investigations has correspondingly been published during the past 2-3 decades since the invention of the first wind lidar in the late 60'ties, including theoretical, numerical and experimental investigations of these wind lidars sampling volume effects on measured mean wind and turbulence profiles.

Major contributions to both cw lidars and to pulsed lidars wind measurement resolutions have over the years been contributed to by both Russian scientists (I.N. Smalikho, V.A. Banakh), German lidar experts (e.g. Chr. Werner, F. Köpp and S. Rahm at DLR Oberpfaffenhofen) and by several American universities and laboratories (e.g. R. Frehlich, B. Banta, T.R. Lawrence, S. Clifford, R.M. Huffaker, C.M. Sonnenschein and F.A. Horigan).

The understanding of the two types of wind lidars spatial resolution applied in this paper refers back to the substantial scientific work previously developed and published by many of these authors.

## 2. On wind lidars sounding volumes

The cw wind lidar:

A mathematical function that characterizes the spatial resolution of a cw wind lidar has been developed by several independent authors including, in order of appearance: Sonnenschein and Horrigan(1971) [1], Lawrence et al.( 19 72) [2] and Smalikho(1995) [3]

Throughout and consistently, the cw wind lidar's radial weighting function is related to the transmitted radiation intensity along the idealized lidars focussed laser beam axis. For Gaussian beams Smalikho(1995) [3] derives the characteristic function  $Q_S(z')$  that describe an ideal cw lidar's radial wind speed resolution along the probing beam axis:

$$Q_S(z') = \frac{1}{\pi k a_0'^2 \left\{ \left(1 - \frac{z'}{R}\right)^2 + \frac{z'^2}{(k a_0')^2} \right\}} \quad (1)$$

Here,  $a_0'$  is the effective radius in the transverse plane of the transmitting telescope (defined by the radius where the intensity has dropped to  $e^{-1}$ ),  $k$  is the wave number, and  $R$  is the cw lidars fixed measurement range determined by its set focus. To a first order approximation  $Q_S(z')$  can be approximated by a Lorentz distribution function, centred at the focal point, see Fig. 1.

$$Q_{S_{cw}}(z'') = \frac{1}{\pi} \frac{z_R}{z_R^2 + (z'')^2} \quad (2)$$

where  $z'' = z' - R$  and where  $z_R$  denotes the Rayleigh length defined as  $z_R = \lambda/\pi R^2/a_0'^2$ . Here,  $a_0$  is defined as the radius where the transverse intensity has dropped to a value  $e^{-2}$ , hence  $a_0 = \sqrt{2} a_0'$ . The quantity  $2z_R$  denotes the optical systems focal depth and also a cw lidar's Full Width Half Maximum *FWHM*. Hence, for the ideal cw lidar, we predict:

$$FWHM_{CW\ Lidar} = 2 z_R \quad (3)$$

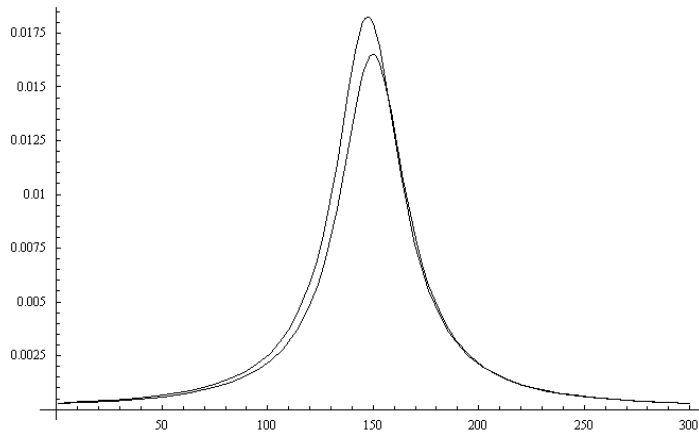


Fig.1: The radial resolution profile of a cw lidars sounding volume: The theoretical curve Eqs.(1) (outer curve), the often used Lorentzian approximation Eqs. (2) (inner curve).

The pulsed wind lidar:

The Doppler signal in a coherent pulsed wind lidar is generated from a sequence of single transmitted pulses. Consequently, the overall range resolution is influenced by both the spatial extent  $\Delta r [m]$  of the transmitted pulse itself and the distance  $\Delta p [m]$  that the pulses travel with the speed of light  $c$  during the range gate sampling time  $\tau$  [4]. The pulsed wind lidars radial spatial resolution has earlier been investigated thoroughly in the literature, see e.g. [4] [5] [6]. Banakh and Smalikho (1997) [5] derives from first principle radar electromagnetic propagation theory an expansion for the pulsed coherent lidar's radial wind velocity estimate  $V_{Pulsed\ Lidar}$ , and show it can be expressed as a convolution of the transmitted laser pulse intensity function  $P_T(t)$  with the sampling time gate window  $\tau [s^{-1}]$ , viz.:

$$V_{Pulsed\ Lidar} = \int_{-\infty}^{\infty} dt P_T(t) \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} dt' V_r \left[ R + \frac{c}{2}(t+t') \right] \quad (4)$$

Here,  $R[m]$  is the range to the centre of the sampling gate and  $V_r [ms^{-1}]$  is the instantaneous radial wind velocity. A Gaussian shaped sounding pulse is often assumed for the transmitted laser pulse:  $P_T(t) = \frac{1}{\sqrt{\pi\sigma_{e^{-1}}}} e^{-t^2/\sigma_{e^{-1}}^2}$  where  $\sigma_{e^{-1}} = t_p$  is the pulse duration in time [s] so that  $\Delta r \sim 2t_p c$  is

the pulse width in space and  $t_p$  is defined via  $P_T(t_p)/P_T(0) = e^{-1}$ . Inserting in Eqs (4) and evaluating the inner integral leads to

$$\langle V_{Pulsed\ Lidar} \rangle = \int_{-\infty}^{\infty} dz' Q_S(z') V_r(r') \quad (5)$$

where

$$Q_{S_{Pulsed\ Lidar}}(z') = \frac{1}{\tau c} \left[ \text{Erf} \left( \frac{2}{ct_p} (z' - R) + \frac{\tau}{2t_p} \right) - \text{Erf} \left( \frac{2}{ct_p} (z' - R) - \frac{\tau}{2t_p} \right) \right] \quad (6)$$

and where  $Q_{S_{Pulsed\ Lidar}}$  is a function that describes the radial resolution of the sounding volume.

By defining the radial width of the sounding volume:  $\Delta z = \int_{-\infty}^{\infty} dz' Q_S(z') / Q_S(R) = Q_S^{-1}(R)$

Banakh and Smalikho (1997)[5] find that the pulsed lidar's effective radial sounding volume size to

$$\text{be } \Delta z = \frac{c\tau}{2} \text{Erf}(\tau/2t_p) \quad (7)$$

an expression that encompasses both the effect of the effective pulse length ( $ct_p/2$ ) and the effective distance the pulse probes ( $c\tau/2$ ) during the gate sampling time  $\tau$ . The corresponding theoretical expression for the Full Width Half Maximum width of a pulsed lidar can from Eqs.(6) be calculated to be:

$$FWHM_{Pulsed\ Lidar} = 0.95 \Delta z \quad (8)$$

### 3. On wind lidars temporal resolution

A continuous wave lidars temporal resolution is limited only by the lidars sampling time  $\tau$  which in turn is limited only by the time-bandwidth product and the speed of the data acquisition hard and software. This enables a 100% duty cycle and hence optimal utilisation of sampling time with a given signal-to-noise ratio in the backscattered signals. The cw lidars sampling time  $\tau$  is typically set with respect to obtaining a high frequency resolution  $1/\tau$  in the measured Doppler spectra.

A pulsed lidars duty cycle is in on the other hand limited by both the sampling time plus the time of flight of the pulse back and forth to the maximum measurement range  $R_M$ :  $\tau + 2R_M/c$ . The duty cycle of a pulsed lidar is therefore limited to  $1/(1 + 2R_M/c\tau) \cdot 100\%$  compared to a cw lidar with identical spectral resolution.

The temporal resolution with a cw lidar is therefore often one or two orders of magnitude larger than obtainable from a pulsed lidar. On the contrary, the pulsed lidar can measure in multiple range gates ( 5 - 10) simultaneously, during the time where a cw lidar can only measure from a single range at a time, set by its focus.

### 4. The cw wind lidar ZephIR and the pulsed wind lidar WindCube

Two new wind lidars have recently become available on the wind energy market for ground-based vertical mean wind and turbulence profiling. The two lidars investigated here are:

- a) The continuous wave ZephIR wind lidar produced by National Power U.K.  
and
- b) the pulsed WindCube WLR7 produced by Leosphere, Fr.

These wind lidars temporal and spatial resolution properties are next to be investigated:

#### Temporal resolution properties:

##### 1) The cw wind lidar "ZephIR":

The ZephIR wind lidar, initially produced by QinetiQ Ltd; Malvern, UK, is now available from National Power Ltd, GB. Vertical wind profile measurements can be obtained in the vertical interval between 10 m and 150 m height above ground. The ZephIR's maximum temporal resolution from a single height is 1 Hz for wind speed and direction determined from the software-build-in Velocity Display Azimuth (VDA) scanning and fitting procedures, and a 1 Hz TKE-turbulence parameter estimated from 50 measurement points per second at a given preset height. Wind vectors at several heights can be measured sequentially but this results in a correspondingly lower over-all data acquisition frequency.

Internally, a ZephIR calculates 200,000 spectra per second using a sampling time  $\tau = 5\mu s$ . In a special research version (Windscanner ZephIR) real-time fast streaming with up to 500 Hz throughput has been obtained with a modified ZephIR at Risø DTU..

## 2) The pulsed wind lidar “Wind Cube”.

Also a pulsed wind lidar is now available from the French company Leosphere. The WindCube pulsed lidar has proven able to measure vertical mean wind profiles up to ~ 300 meters above ground during favourable conditions. Its temporal resolution for acquiring a full 3-D wind vector, obtained from four orthogonal measurement points, is about 1/6 Hz (0.16 Hz). As the wind vectors can be extracted from three out of these four radial measurement points in the scan sequence, a wind vector can be deduced every 1/ 4.5 Hz ( 0.22 Hz). However, being a pulsed system, all mean wind speed, direction and turbulence data can be acquired simultaneously from several (e.g. 5) heights or range gates simultaneously.

Internally in a WindCube, 10,000 spectra are recorded in standard mode over a  $\frac{1}{2}$  s acquisition time. With a (standard) WindCube sampling time of only  $\tau = 0.2 \mu s$  the resulting Doppler spectra become 25 times sparser compared to Doppler spectra velocity resolution within a standard ZephIR. A dedicated maximum likelihood spectral estimator is applied for fitting a model spectrum to the observed Doppler peak finding and to minimize the measurement uncertainty. Throughput of 5 -10 Hz simultaneously from multiple ranges should in principle be possible from a WindCube.

### Sounding volumes and spatial resolution properties

As seen, the two lidar types, cw and pulsed, have also very different spatial resolution properties. This will next be investigated quantitatively in light the theoretical discussion of sounding volumes in chapter 2. Meanwhile, experimental evaluations are going in connection with the [Windscanner.dk](http://Windscanner.dk) research Infrastructure presently under development at Risø DTU, with the purpose to evaluate practical obtainable performances for wind lidars to be designed for a new Danish lidar-based Windscanner research infrastructure, an open research facility which success in particular will be dependent on obtaining the best possible performances with the two different lidar types.

### Cw ZephIR:

The CW system range-resolution is determined by the cw lidar’s Rayleigh length which increases with the square of the range to the measurement point, cf. Eqs. (3). A cw ZephIR lidars range resolution is characterized by its radial effective probe length,  $\Delta z_{ZephIR} \sim \text{FWHM}$  (Full Width Half

Maximum), which relates to the Rayleigh length  $z_R = \frac{\lambda R^2}{\pi a_0^2}$  as  $\Delta z_{ZephIR} = 2z_R$  for a Gaussian

beam focused within the far-field limit  $D_{\text{Far-Field}} \sim \pi \frac{a_0^2}{\lambda}$ . At the focal point the beam waist radius has

its minimum:  $w_0 = \frac{\lambda R}{\pi a_0}$ . The Rayleigh length and hence  $\Delta z_{ZephIR}$  increases quadratically with

sensing range. When measuring at short range, the probe length is only a few decimetres. At the 100m range the ZephIR’s Rayleigh length  $z_R$  has been estimated from the ZephIR’s 3” 200 mm focal length optical lens and a corresponding estimated effective aperture radius of  $a_0 = 24$  mm, defined as where the intensity has dropped by a factor  $e^{-2}$  of its center value. The Rayleigh length has here at 100 m range have been estimated to be [7] ~ 8.5 m:

$$z_{R\_ZephIR} \sim 0.00085R^2 \quad @ \quad a_{0\_ZephIR} \approx 24 \text{ mm} \quad (9)$$

At a measurement range  $R$  of 100 m, the  $FWHM_{\text{ZephIR}} @ 100 \text{ m}$  can consequently be estimated to be of the order of  $\sim 17.0 \text{ m}$ . For a range of 150 m, the corresponding radial dimension of the sounding volume can be estimated to be:

$$FWHM_{\text{ZephIR}} @ 150 \text{ m is } \sim 38.5 \text{ m.} \quad (10)$$

Evaluations of standard cw ZephIR's Rayleigh length have been investigated experimentally during the MusketeerEx 2007 and MusketeerEx 2008 Høvsøre field trials from hard target measurements. However, measurements performed using solid targets show larger focal volumes [8] due to the different nature of hard target speckle statistics relative to diffuse atmospheric backscatters [9]. Also assessments on the cw lidars atmospheric measured wind speed transfer functions have been investigated experimentally. These measurements seems to confirm the prediction in Eqs. (9) by Harris et al. [9] to be accurate within a factor of 2 [10]. In this study and until further evidence is available, the estimate in Eqs. 9 will therefore be used.

### **Pulsed WindCube**

With unfocussed (collimated) beams the pulsed WindCube's radial range resolution is theoretically determined by the transmitted laser pulse width and the distance the pulse travels during the sampling time per Doppler spectrum acquisition, cf. Eqs. (7).

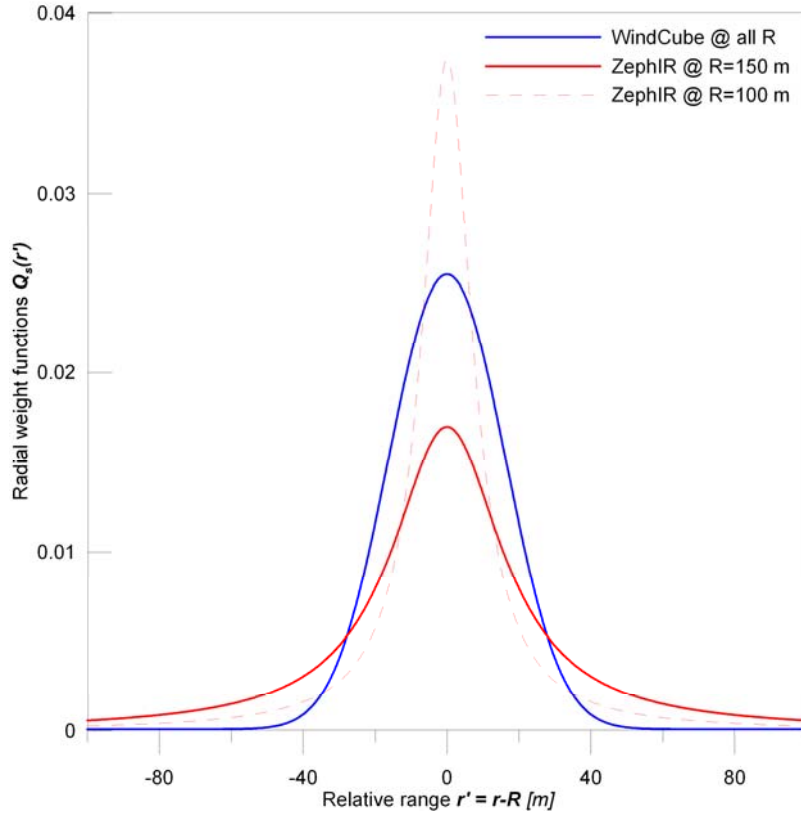
The WindCube's transmitted laser pulse intensity as function of time is approximately Gaussian-shaped and is according to J.P. Cariou, Leosphere (personal communication) approximately  $\sim 200 \text{ ns}$  in terms of  $FWHM$  (i.e., corresponding to 60 m long pulses). The WindCube's corresponding pulse time parameter (cf. chapter 2):  $\sigma_{e^{-1}} = t_p$  is hence estimated to be of the order of  $\sim 120 \text{ ns}$ . The standard WindCube WLS7's range gates sampling time  $\tau$  is factory preset to 200 ns.

Based on these two parameters for a standard WindCube the following estimate can now be obtained from Eqs. (8):

$$FWHM_{\text{WindCube}} \approx 37.5 \text{ m.} \quad (10)$$

## 5. Discussions

### Wind Lidar Spatial Resolution: CW ZephIR vs. pulsed WindCube



**Fig. 2:** The cw ZephIR and the pulsed WindCubes sounding volumes radial wind speed resolution functions intercompared at fixed measurement range  $R=150$  m. The stippled curve represent the ZephIR's radial sounding volumes resolution at range  $R = 100$  m.

Fig 2. shows the standard ZephIR's predicted sounding resolution calculated from Eqs. (2) with the above estimated parameters at range 150 m, plotted together with the corresponding WindCubes spatial resolution function calculated from Eqs (6). While the *FWHM* of the cw ZephIR increases with the square of the range the corresponding *FWHM* of the WindCube is (for un-collimated beams) assumed constant at all ranges.

The range to equal size  $FWHM_{ZephIR} \approx FWHM_{WindCube}$  can be calculated to occur at  $R \sim 148.5$  m (150 m), cf. the curves plotted in Fig.2. This radial range corresponds to a vertical measurement height of  $\sim 130$  m above ground, with a 30 degree azimuth angle scanning cones.

Fig. 2 also reveals a significant different between the two lidar types radial sounding resolutions. Although the ZephIR and the WindCube theoretically have similar *FWHM* at range 150 m, the cw lidar has significant longer tails compared to the tails of the pulsed lidar. While the tails of the cw ZephIR's Lorentzian tapers off radially as  $\sim r^{-2}$  the tails of the pulsed WindCube lidar trade off as  $\sim e^{-r'^2}$ , that is, much faster.

## 6. Sounding volumes effect on measured mean wind profiles:

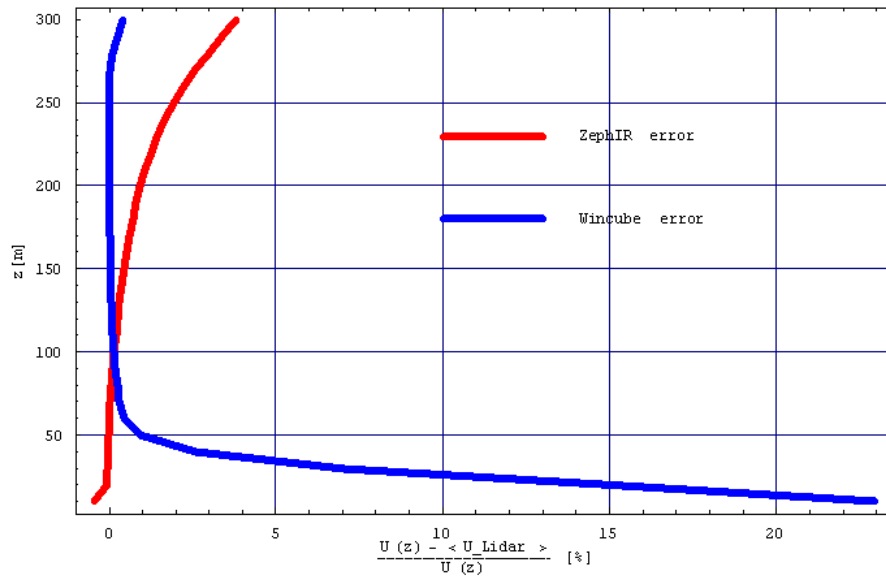


Fig.3: Prediction of the relative sounding volume induced relative errors on lidar-measured neutral mean wind profiles, as function of range  $z$ . The corresponding relative wind speed measurement error as function of height using a 30 degree wedge can be obtained by multiplying the ordinate with  $\sqrt{3}/2 \sim 0.87$ .

Fig. 3 shows the relative measurement error anticipated due to finite sounding volumes of both the ZephIR and the WindCube for wind speed profile measurements under neutral atmospheric conditions. While the WindCube is seen to underestimate the wind speed significantly at heights lower than 50 meters the ZephIR begins to underestimate the wind speeds by more than 1 % starting at heights 200 m and above.

### Effects of focussing a pulsed WindCube

Numerical investigations have also been undertaken to examine the effects of slightly focusing the WindCube. The corresponding Rayleigh length for a focussed WindCube has been estimated from scaling of the Rayleigh length from the 3" optics used in a standard ZephIR down to the 2" standard lenses used with a standard WindCube (i.e. scaling the Rayleigh length by the aperture ratio squared  $(3/2)^2$ ), hence the Rayleigh length of a standard WindCube, focussed to 100 m range has been estimated to be of the order of  $\sim 38$  m. While this significantly reduces the FWHM of the WindCube (from at 37.5 m to 27 m) at the fixed range gate at  $R=100$  m, it jeopardize on the other hand the sensitivity at range 150 m to below 20 % of the corresponding unfocused system.

Focussing the WindCube to range 150 meters, on the other hand, only halves the sensitivity at the 100 m range, which is ok to sacrifice with the "one over R squared" S/N sensitivity dependency of a pulsed system", but it also only reduces the overall FWHM of the combined focussed pulsed system in this investigation to  $\sim 33$  m.

### Turbulence:

Wind lidar's non-negligible sounding volumes have also significance for the interpretation of turbulence as measured both by cw and by pulsed lidars, cf. e.g. the recent investigation in [11].

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